LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – NOVEMBER 2022

PMT 3504 – ALGORITHMIC GRAPH THEORY

Date: 09-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

ANSWER ALL QUESTIONS:

1. (a) Prove that a self-complementary graph has either 4n or 4n+1 vertices.

(or)

(b) Show that the girth of the petersen graph is 5. **(5)**

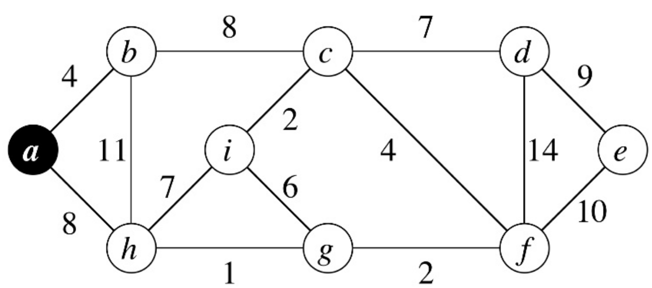
(c) i) Define bipartite garph and prove that every bipartite graph has no odd cycles.

ii) Prove that a connected graph G is Eulerian if and only if all of its vertices have even degrees. **(7+8)**

(or)

d) i) Write Dijkstra's algorithm.

ii) Find the shortest path from the source node “a” to all other nodes using Dijkstra's algorithm for the following graph: **(5+10)**



2. (a) Show that the closure of a graph is well defined.

(or)

(b) State and prove Euler’s formula for planar graph **(5)**

(c) i) State and prove Ore’s theorem.

ii) State and prove Dirac’s theorem. **(5+10)**

(or)

(d) State and prove Kuratowski’s theorem. **(15)**

3. (a) Define perfect elimination ordering with an example.

(or)

(b) Write transitive orientation property and give an example. **(5)**

(c) i) Prove that every triangulated graph has a simplicial vertex. If G is not a clique then prove that it has two non-adjacent simplicial vertices.

ii) Show that every induced subgraph of a chordal graph is chordal. **(9+6)**

(or)

(d) i) Write breadth first search algorithm.

ii) State and prove any three properties of split graphs. **(5+10)**

4. (a) Draw a permutation graph for $(\mathbf{4}, \mathbf{3}, \mathbf{5}, \mathbf{2}, \mathbf{7}, \mathbf{6}, \mathbf{1})$.

(or)

(b) Give an example that representation of a split graph need not be unique. **(5)**

(c) i) Prove that permutation graph is a comparability graph.

ii) Prove that if $\boldsymbol{G}$ is a permutation graph, then $\overline{\boldsymbol{G}}$ is a permutation graph.

iii) Define permutation labeling with an example. **(5+5+5)**

(or)

(d) Prove that C_7 is not comparability graph. Give necessary and sufficient condition for a tree to be a split graph. **(15)**

5. (a) Define a circular arc graph with an example.

(or)

(b) Define an aestroidal triple with an example. **(5)**

(c) i) Show that an interval graph satisfies triangulated property. Also discuss about its converse.

ii) Prove that the complement of an interval garph satisfies the transitive orientation property. **(9+6)**

(or)

(d) i) State and prove characterisation theorem for circular arc graph.

ii) Give an example to show that circular arc graph need not be an interval graph. **(10+5)**

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